Name Key **Pre-Calculus Final Exam Review #1**

Topics for Precalculus Midterm 2020: **My Seat/Location:** ______

Basic Trig - Δs, unit circle, evals.
Graphing
Identities
Eq'ns
Apps.

1) Find the exact value of $\cos\frac{\pi}{12}$. $= 15^\circ$

$(45^\circ - 30^\circ)$

$\cos(45-30) = \cos 45 \cos 30 + \sin 45 \sin 30$

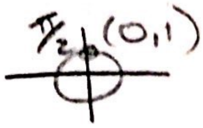
$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

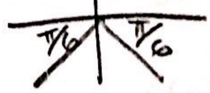
$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

2) Solve for all values of x in the interval $0 \le \theta < 2\pi$.

$2\sin^2\theta - \sin\theta - 1 = 0$

$\sin\theta = \frac{1 \pm \sqrt{1-(4\cdot 2\cdot -1)}}{4} = \frac{1 \pm \sqrt{9}}{4} < \begin{matrix} \frac{1+3}{4} = 1 \\ \frac{1-3}{4} = -\frac{1}{2} \end{matrix}$





$\boxed{\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

3) Solve for all values θ in the interval $0 \le \theta < 2\pi$.

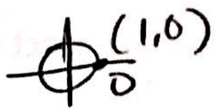
$2\sin^2\theta + 3\cos\theta - 3 = 0$

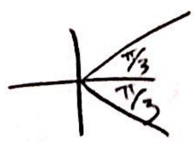
$2(1-\cos^2\theta) + 3\cos\theta - 3 = 0$

$2 - 2\cos^2\theta + 3\cos\theta - 3 = 0$

$2\cos^2\theta - 3\cos\theta + 1 = 0$

$\cos\theta = \frac{3 \pm \sqrt{9-(4\cdot 2\cdot 1)}}{4} = \frac{3 \pm \sqrt{1}}{4} < \begin{matrix} \frac{4}{4} = 1 \\ \frac{2}{4} = \frac{1}{2} \end{matrix}$





$\boxed{\theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}}$

4) What is $\frac{\tan x}{\sec x}$ expressed in simplest form?

$$\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{\sin x}{\cancel{\cos x}_1} \cdot \frac{\cancel{\cos x}^1}{1} = \boxed{\sin x}$$

5) The expression $\frac{1-\cos^2 x}{\sin^2 x}$ is equivalent

$$\frac{\sin^2 x}{\sin^2 x} = \boxed{1}$$

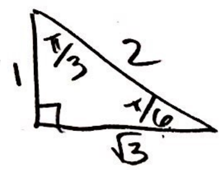
6) The expression $\frac{\sin 2\theta}{\sin^2\theta}$ is equivalent to

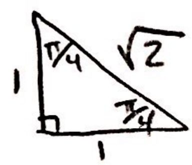
$$\frac{2\cancel{\sin\theta}\cos\theta}{\sin^{\cancel{2}}\theta} = \frac{2\cos\theta}{\sin\theta} = \boxed{2\cot\theta}$$

7) The expression $(1+\cos x)(1-\cos x)$ is equivalent to

$$1-\cos^2 x$$

$$\boxed{\sin^2 x}$$





8) a) Find the exact value of $Tan\left(\frac{5\pi}{12}\right)$ $=75^\circ$

$$\tan(30+45) = \frac{\tan 30 + \tan 45}{1-\tan 30\tan 45} = \frac{\frac{1}{\sqrt{3}} + \frac{1\cdot\sqrt{3}}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)(1)}$$

$$= \frac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}\cdot 1}{\sqrt{3}\cdot 1} - \frac{1}{\sqrt{3}}} = \frac{\frac{1+\sqrt{3}}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}} = \boxed{\frac{1+\sqrt{3}}{\sqrt{3}-1}}$$

b) Find the exact value of $Sin\left(\frac{\pi}{12}\right)$ $=15^\circ$

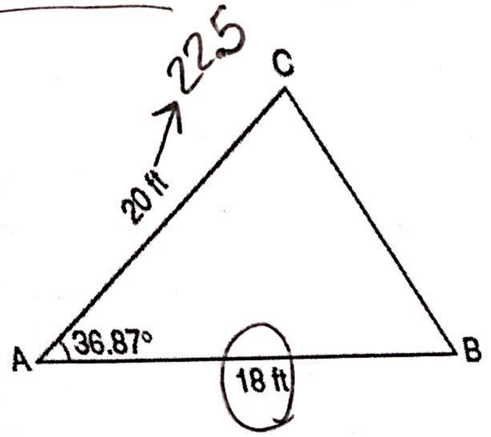
$$\sin(45-30)$$

$$=\sin 45\cos 30 - \cos 45\sin 30$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

9)

A) The accompanying diagram shows a triangular plot of land that is part of Mrs. Dounias' garden. She needs to change the dimensions of this part of the garden, but she wants the area to stay the same. She increases the length of side AC to 22.5 feet. If angle A remains the same, by how many feet should side AB be *decreased* to make the area of the new triangular plot of land the same as the current one?



$A_{current} = \frac{1}{2}(20)(18)\sin 36.87$

$= 108$

$108 = \frac{1}{2}(22.5)x \cdot \sin 36.87$

$108 = 6.75x$

$x = 16$

$18 \rightarrow 16$ dec. by $\therefore$ 2 ft.

B) If the terminal side of an angle of θ radians passes through the point $\left(\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right)$ on a unit circle, find the value of $\tan\theta$.

x, y

$(\cos\theta, \sin\theta)$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-3}{\cancel{\sqrt{10}}}}{\frac{1}{\cancel{\sqrt{10}}}} = \boxed{-3}$

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10) Using $f(x) = -2\sin\left[2\left(x - \frac{\pi}{4}\right)\right] + 2$

a) State the amplitude 2

b) State the period $\frac{2\pi}{2} = \pi$

c) Horizontal shift $\pi/4$ right

xscale $= \frac{\pi}{4}$

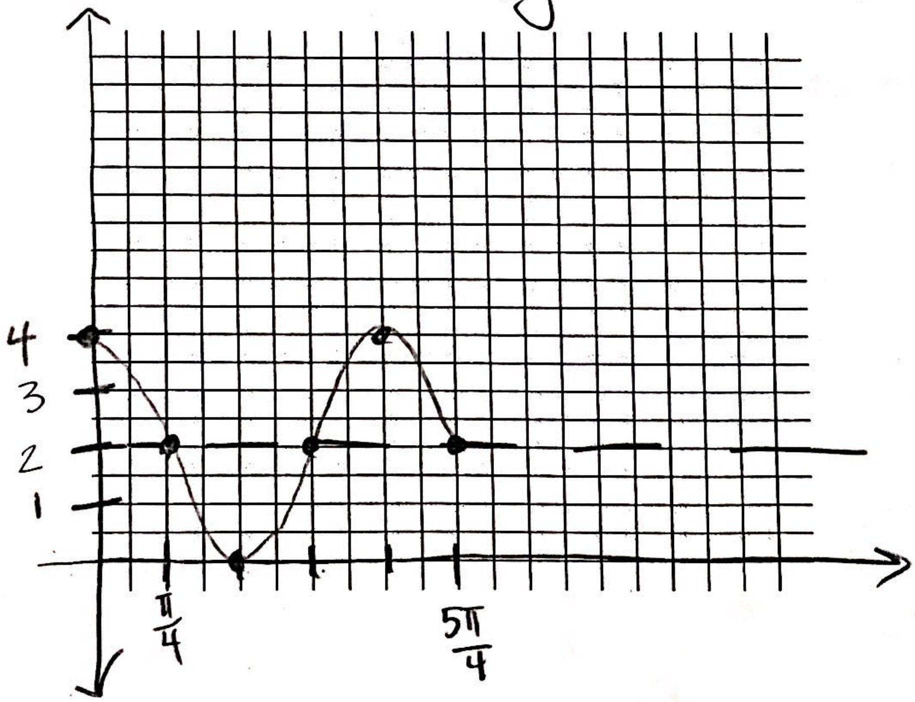
d) Vertical shift 2 up

e) Sketch f(x), in the interval $0 \le x \le \frac{5\pi}{4}$ on the grid below.

$-2+2=0$
$2+2=4$

f) State the ***range*** of the graph, in the given interval, sketched in part e $[0, 4]$

g) Name f(x) as a Cosine curve: $y = 2\cos(2x) + 2$



Name Key

Midterm Review Sheet#2

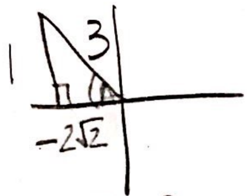
$(1+\sin\theta)(1-\sin\theta)$

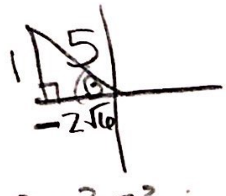
$(1-\sin^2\theta)$

1. Prove the identity: $1-\sin\theta=\frac{\cos^2\theta}{1+\sin\theta}$

$1-\sin\theta=1-\sin\theta$ ✓

2. Given that A and B are in quadrant II, $\sin A=\frac{1}{3}$, and $\sin B=\frac{1}{5}$, find $\cos(A-B)$.





$1^2+x^2=3^2$

$\sqrt{x^2}=\sqrt{8}$

$1^2+x^2=5^2$

$\sqrt{x^2}=\sqrt{24}$

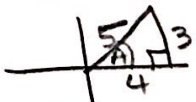
$\cos(A-B)=\cos A\cos B+\sin A\sin B$

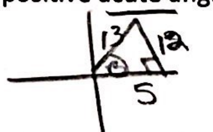
$=\left(\frac{-2\sqrt{2}}{3}\right)\left(\frac{-2\sqrt{6}}{5}\right)+\left(\frac{1}{3}\right)\left(\frac{1}{5}\right)$

$=2\sqrt{3}$

$\frac{4\sqrt{12}}{15}+\frac{1}{15}=\boxed{\frac{8\sqrt{3}+1}{15}}$

3. If $\sin A=\frac{3}{5}$ and $\sin B=\frac{12}{13}$ where A and B are positive acute angles, find:
   a) $Sin(A-B)$
   b) $Tan(A+B)$





a) $\sin A\cos B-\cos A\sin B$

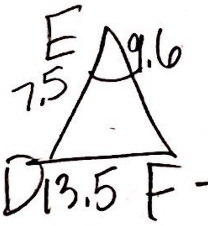
$\left(\frac{3}{5}\right)\left(\frac{5}{13}\right)-\left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$

$\frac{15}{65}-\frac{48}{65}=\boxed{\frac{-33}{65}}$

b) $\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\frac{3}{4}+\frac{12}{5}}{1-\left(\frac{3}{4}\right)\left(\frac{12}{5}\right)}$

$=\frac{\frac{15}{20}+\frac{48}{20}}{1-\frac{36}{20}}=\frac{\frac{63}{20}}{\frac{-16}{20}}=\boxed{\frac{63}{-16}}$

4. Find, to the nearest minute, the measure of the largest angle of ΔDEF if DE = 7.5, EF=9.6, DF = 13.5



$13.5^2=7.5^2+9.6^2-2(7.5)(9.6)\cos E$

$182.25=148.41-144\cos E$

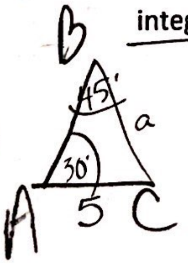
$-148.41\quad -148.41$

$\frac{33.84}{-144}=\frac{-144\cos E}{-144}$

$-.235=\cos E$

$\boxed{\angle E=103°35'}$

5. Given Triangle ABC, angle A = 30, angle B = 45 and b = 5. Find the length of side a to the nearest integer.



$\frac{a}{\sin 30}=\frac{5}{\sin 45}$

$\frac{a\sin 45}{\sin 45}=\frac{5\sin 30}{\sin 45}$

$a=3.5355$

$\boxed{a=4}$

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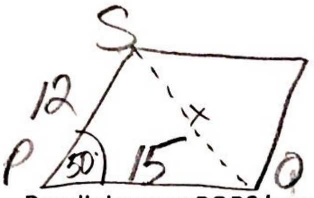


$A = ab\sin C$

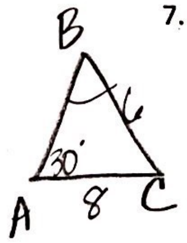
$(12)(15)\sin 50° = 137.8879998$

$\approx \boxed{138}$

6. Parallelogram PQRS has sides with length 12 and 15. The angle between them is 50 degrees. Find the area of the parallelogram. Find the length of the diagonal (round to the nearest tenth).

$x^2 = 12^2 + 15^2 - 2(12)(15)\cos 50°$

$\boxed{x = 11.7}$



7. In triangle ABC, measure angle A = 30, a = 6, and b = 8. How many triangles can be formed? $\boxed{2}$

$\frac{6}{\sin 30} = \frac{8}{\sin B}$

$\frac{\cancel{6}\sin B}{\cancel{6}} = \frac{8\sin 30}{6}$

$\sin B = 2/3$

$\angle B \approx 42°$

OR $138°$

A	30	30
B	42	138
C	108	12

8. If x is a positive acute angle, solve $3(\cot x - 1) = 0$ to the nearest degree.

$3\cot x - 3 = 0$

$\cancel{\tan x}\left(\frac{3}{\cancel{\tan x}} - 3 = 0\right)$

$3 - 3\tan x = 0$

$\frac{3 = 3\tan x}{3 \quad 3}$

$\tan x = 1$

ref. $\angle = 45°$

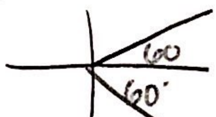
$\boxed{x = 45°}$

9. Solve the equation for one value of y in the interval $0 \leq y < 360$: $2\cos^2 y + \cos y - 1 = 0$.

$(2\cos y - 1)(\cos y + 1) = 0$

$\cos y = 1/2$ | $\cos y = -1$

ref. $\angle = 60°$



$\boxed{y = 60°, 300°, 180°}$

10. The tide of a boat dock can be modeled by the equation $y = -2\cos\left(\frac{\pi}{6}t\right) + 8$ where t is the number of hours past noon and y is the height of the tide, in feet. For how many hours between t = 0 and t = 12 is the tide at least 7 feet?

$7 \leq -2\cos\left(\frac{\pi}{6}t\right) + 8$

$-8 \qquad\qquad\qquad -8$

$\frac{-1 \leq -2\cos\left(\frac{\pi}{6}t\right)}{-2 \qquad -2}$

$+\frac{1}{2} = \cos\left(\frac{\pi}{6}t\right)$

$\boxed{\frac{\pi}{3} \text{ or } \frac{5\pi}{3}}$

$\left(\frac{6}{\pi}\right)\frac{\pi}{6}t = \frac{\pi}{3}\left(\frac{6}{\pi}\right)$ $t = 2$

$\left(\frac{6}{\pi}\right)\frac{\pi}{6}t = \frac{5\pi}{3}\left(\frac{6}{\pi}\right)$

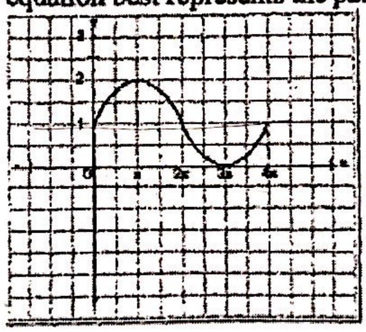
$t = 10$

graph $t = 2 \to 10$

8 hrs.

From $2 \to 10$ 8hrs.

11. In physics class, Eva noticed the pattern shown in the accompanying diagram on an oscilloscope. What equation best represents the pattern shown on this oscilloscope?



$P = 4\pi$ $B = \frac{2\pi}{4\pi}$

$y = 1\sin\left(\frac{1}{2}x\right) + 1$

Name Key

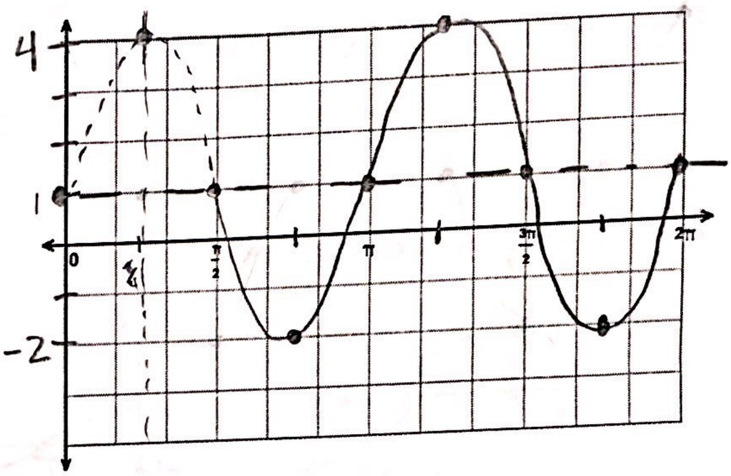
Review #3 for Midterm

1. Identify the following aspects of the trig function below and then graph the function.

$$f(x) = -3\sin 2\left(x - \frac{\pi}{2}\right) + 1, \qquad 0 \le x \le 2\pi$$

a. Amplitude: 3
b. Period: $2\pi/2 = \pi$
c. Phase shift (horizontal): $\frac{\pi}{2}$ right
d. Vertical shift: up 1
e. Express the graphed function as a cosine function:

$y = 3\cos\left(2\left(x - \frac{\pi}{4}\right)\right) + 1$



xscale $= \frac{\pi}{4}$ Max $= 3+1 = 4$
Min $= -3+1 = -2$

2. a) Simplify: $\frac{1}{\cos t} - \sin t \cdot \tan t$

$\frac{1}{\cos t} - \frac{\sin t \cdot \sin t}{\cos t}$

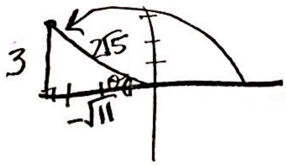
$\frac{1 - \sin^2 t}{\cos t} = \frac{\cos^2 t}{\cos t} = \boxed{\cos t}$

b) Prove: $\frac{\sin^2\theta}{1-\cos\theta} = 1 + \cos\theta$

$1 - \cos^2\theta$

$= \frac{(1+\cos\theta)\cancel{(1-\cos\theta)}}{\cancel{1-\cos\theta}} = 1 + \cos\theta$ ✓

3. The terminal side of an angle of θ radians passes through the point $(-\sqrt{11}, 3)$. Find the value of all 6 trig functions.



$3^2 + (-\sqrt{11})^2 = c^2$
$9 + 11 = c^2$
$\sqrt{20} = \sqrt{c^2}$
$c = 2\sqrt{5}$

$\sin\theta = \frac{3}{2\sqrt{5}}$ $\csc\theta = \frac{2\sqrt{5}}{3}$

$\cos\theta = -\frac{\sqrt{11}}{2\sqrt{5}}$ $\sec\theta = \frac{2\sqrt{5}}{-\sqrt{11}}$

$\tan\theta = \frac{3}{-\sqrt{11}}$ $\cot\theta = -\frac{\sqrt{11}}{3}$

4. Solve for all real values of $0 \le x \le 2\pi$ in radian measure: $2\cos^2 x - \sin x = 1$

$2(1 - \sin^2 x) - \sin x - 1 = 0$
$-2\sin^2 x - \sin x + 1 = 0$
$2\sin^2 x + \sin x - 1 = 0$

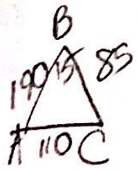
$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ $\sin x = -1$

ref. $\angle = \frac{\pi}{6}$ $\frac{3\pi}{2}$ $(0,-1)$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

5. In ΔABC, a = 85 ft, b = 110 ft, and c = 190 ft.



a) Find the measure of angle B to the nearest degree.

$110^2 = 85^2 + 190^2 - 2(85)(190)\cos B$
$12100 = 43325 - 32300\cos B$
$-31225 = -32300\cos B$
$.9667 = \cos B$
$\angle B = 14.82$
$\boxed{15^\circ}$

b) Use the measure of angle B to find the area of ΔABC to the nearest square foot.

$A = \frac{1}{2}(190)(85)\sin 15^\circ = 2089.96$

$\boxed{2090}$

Name ______________________

6. Write down a simpler expression that $\sin(\pi - x)$ is equivalent to: $(-1, 0)$

$\sin\pi\cos x - \cos\pi\sin x$

$(0)\cdot\cancel{\cos x} - (-1)\cdot\sin x$

$= \boxed{\sin x}$

7. Solve for all values of θ in the interval $0 \le \theta < 2\pi$: $\sqrt{3}\cot\theta + 3 = 0$

S A T

$\pi/6$ $\pi/6$

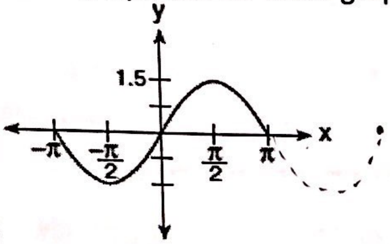
$\frac{\sqrt{3}\cot\theta}{\sqrt{3}} = \frac{-3}{\sqrt{3}}$

$\tan\theta = \frac{-\sqrt{3}}{3}$

$\theta = \pi/6$

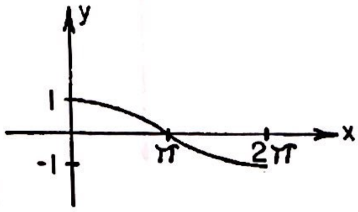
$\boxed{\theta = \frac{5\pi}{6}, \frac{11\pi}{6}}$

8. Find an equation for these graphs:



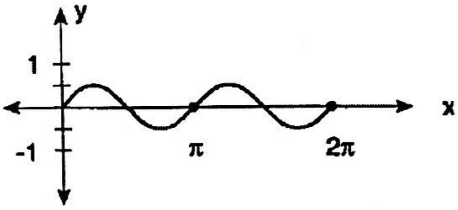
a)

$y = 1.5\sin x$



b)

$y = \cos\left(\frac{1}{2}x\right)$



c)

$y = \frac{1}{2}\sin(2x)$

9. Let $\sec x = -2$.

a) What is $\cos x$? $-\frac{1}{2}$

b) Solve the equation above for $x \in [0, 2\pi)$

$\cos x = -\frac{1}{2}$

ref. $x = \frac{\pi}{3}$

$\pi/3$ $\pi/3$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$

10. Consider the equation $\sqrt{\cos^2 x} = \pm\sqrt{\frac{3}{4}}$.

a) How many solutions do you expect this equation to have, for $x \in [0, 2\pi)$? Why?

4, 2 pos. + 2 neg.

b) Find those solutions!

$\cos x = \pm\frac{\sqrt{3}}{2}$

ref. $x = \frac{\pi}{6}$

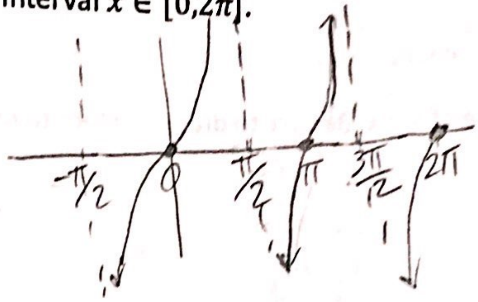
$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

Name Key

11. Consider the function $f(x) = \tan x$ on the interval $x \in [0,2\pi]$.

a) What is the period of f?

π



b) What are the zeros of f?

$0, \pi, 2\pi$

c) Where does f have vertical asymptotes?

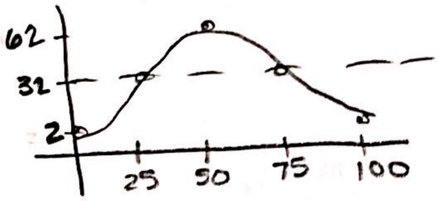
$\pi/2, 3\pi/2$

$P = 100$ $Xscl = \frac{100}{4} = 25$

12. Sognefjord is going on a Ferris wheel. Its diameter is 60 feet, and it takes 100 seconds to complete one full counterclockwise rotation. Sognefjord enters the wheel at its lowest point, which is 2 feet off of the ground, when $t = 0$.

Min. 2 $C: \frac{64}{2} = 32$
Max 62

a) Sketch one cycle of the ride. Label important points on the x and y axes.



$B = \frac{2\pi}{100} = \frac{\pi}{50}$

b) Write an equation of the form $f(t) = A\cos(Bt) + C$ to model Sognefjord's height above the ground t seconds after she started the ride. Then check in your calculator to make sure your equation matches your graph from a!

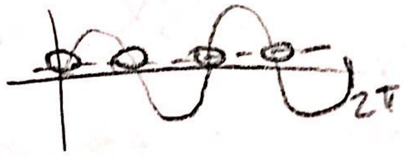
$f(t) = -30\cos\left(\frac{\pi}{50}t\right) + 32$

13. How many solutions do you expect the following equations to have, for $x \in [0,2\pi)$? Why? No need to solve.

graph

a) $\sin(x) = \frac{1}{3}$ 2

b) $\sin(2x) = \frac{1}{3}$

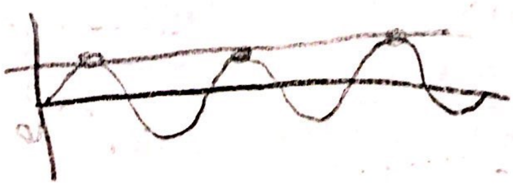


4

c) $\sin(x) = 3$ 0

grtr. than 1.

d) $\sin(3x) = 1$

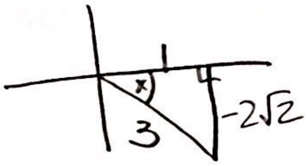


3

Name ______________________

14. Let $\cos x = \frac{1}{3}$ where x terminates in quadrant 4.

a) Find the exact value of $\sin x$. Be sure to draw a picture to make sure your answer makes sense!



$1^2 + x^2 = 3^2$

$1 + x^2 = 9$

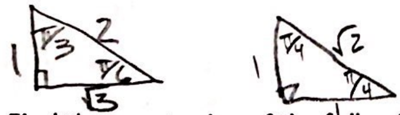
$\sqrt{x^2} = \sqrt{8}$

$x = 2\sqrt{2}$

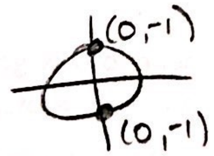
$\sin x = \frac{-2\sqrt{2}}{3}$

b) Find the exact value of $\sin(2x)$. $\longrightarrow$ double ∡

$2\sin x \cos x = \frac{2}{1}\left(\frac{-2\sqrt{2}}{3}\right)\left(\frac{1}{3}\right) = \boxed{\frac{-4\sqrt{2}}{9}}$



15. Find the exact value of the following:



a) $\sin\left(\frac{2\pi}{3}\right)$ $\frac{\sqrt{3}}{2}$

b) $\cos\left(\frac{11\pi}{6}\right)$ $\frac{\sqrt{3}}{2}$

c) $\sin\left(\frac{7\pi}{4}\right)$ $-\frac{\sqrt{2}}{2}$

d) $\cos\left(\frac{3\pi}{2}\right)$ 0

e) $\tan\left(\frac{5\pi}{6}\right)$ $-\frac{\sqrt{3}}{3}$

f) $\cot\left(\frac{\pi}{2}\right)$ 0

g) $\sec\left(\frac{\pi}{6}\right)$ $\cos 30^\circ = \frac{\sqrt{3}}{2}$ $\frac{2}{\sqrt{3}}$

h) $\csc\left(\frac{\pi}{4}\right)$ $\sin 45 = \frac{\sqrt{2}}{2}$ $\frac{2}{\sqrt{2}}$

16. Prove that:

a) $\frac{\sec x}{\cot x + \tan x} = \sin x$

$$\frac{\frac{1}{\cos x}}{\frac{\cos x \cdot \cos x}{\cos x \cdot \sin x} + \frac{\sin x \cdot \sin x}{\cos x \cdot \sin x}} = \frac{\frac{1}{\cos x}}{\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}} = \frac{1}{\cos x} \cdot \frac{\sin x \cancel{\cos x}}{\cancel{\cos^2 x + \sin^2 x}} = \sin x \checkmark$$

b) $\frac{1 + \cot x}{\tan x + 1} = \cot x$

$$\frac{\frac{\sin x \cdot 1}{\sin x \cdot 1} + \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \frac{1 \cdot \cos x}{1 \cdot \cos x}} = \frac{\frac{\sin x + \cos x}{\sin x}}{\frac{\sin x + \cos x}{\cos x}} = \frac{\cancel{\sin x + \cos x}}{\sin x} \cdot \frac{\cos x}{\cancel{\sin x + \cos x}} \checkmark$$

$= \frac{\cos x}{\sin x} = \cot x$

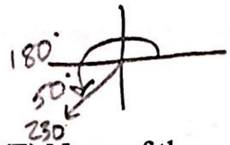
Precalculus Name Key

I. Multiple Choice

C 1. If sin θ < 0 and tan θ > 0, then in which quadrant does θ lie?

(A) I (B) II (C) III (D) IV

C 2. Given an angle of 230°, its reference angle is:



(A) 130° (B) 40° (C) 50° (D) 30° (E) None of these

B 3. The domain of y = sin (x) is:

(A) -90° ≤ x ≤ 90° (B) -∞ < x < +∞ (C) -1 ≤ y ≤ 1
(D) 0 ≤ y ≤ 180° (E) None of these

C 4. The range of y = cos (x) is:

(A) -90° ≤ x ≤ 90° (B) -∞ < x < +∞ (C) -1 ≤ y ≤ 1
(D) 0 ≤ y ≤ 180° (E) None of these

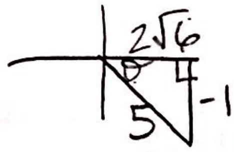
C 5. The exact value of $\csc\left(\frac{5\pi}{3}\right)$ is

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\csc = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

(A) $\frac{-\sqrt{3}}{2}$ (B) $\frac{2\sqrt{3}}{3}$ (C) $-\frac{2\sqrt{3}}{3}$ (D) 2 (E) None of these

D 6. Given that $\sin\theta = -\frac{1}{5}$ and tan θ < 0, determine the value of cos θ.



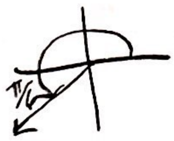
(A) $-\frac{\sqrt{26}}{5}$ (B) $\frac{\sqrt{26}}{5}$ (C) $-\frac{2\sqrt{6}}{5}$ (D) $\frac{2\sqrt{6}}{5}$ (E) None of these

$x^2+(-1)^2=5^2$

$x^2+1=25$

$\sqrt{x^2}=\sqrt{24}=2\sqrt{6}$

A 7. Determine the exact value of $\sin\left(\frac{7\pi}{6}\right)$



(A) $-\frac{1}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) None of these

C 8. Determine the amplitude of y = 3 sin (2x) + 4

(A) 1 (B) 2 (C) 3 (D) 4 (E) None of these

Part 2:

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Convert from degrees to radians.

1) $210° \cdot \frac{\pi}{180}$ 　　1) C

A) $\frac{7\pi}{12}$　　B) $\frac{7\pi}{3}$　　C) $\frac{7\pi}{6}$　　D) $\frac{7\pi}{5}$

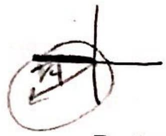
* Convert the radian measure to degree measure. Use the value of π found on a calculator and round answers to two decimal places.

2) $\pi/2 \cdot \frac{180}{\pi}$ 　　2) D

A) 1.57°　　B) (π/2)°　　C) 90π°　　D) 90°

Find the value of the unique real number θ between 0 and 2π that satisfies the given conditions.

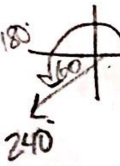
4) $\cos\theta = -\frac{\sqrt{2}}{2}$ and $\tan\theta > 0$ 　　4) B



A) $\frac{3\pi}{4}$　　B) $\frac{5\pi}{4}$　　C) $\frac{2\pi}{3}$　　D) $\frac{\pi}{4}$

Evaluate without using a calculator by using ratios in a reference triangle.

5) sec 240° 　　5) A

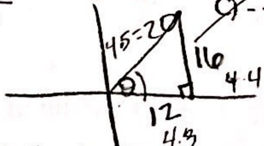


A) -2　　B) $-\frac{2\sqrt{3}}{3}$　　C) 2　　D) $\frac{2\sqrt{3}}{3}$

$\cos 60 = -\frac{1}{2} \to -2$

6) $\tan\left(-\frac{2\pi}{3}\right)$　$\tan 60 = \sqrt{3}$ 　　6) A

A) $\sqrt{3}$　　B) $\frac{\sqrt{3}}{3}$　　C) $-\frac{\sqrt{3}}{3}$　　D) $-\sqrt{3}$



Point P is on the terminal side of θ. Evaluate the six trigonometric functions for θ. If the function is undefined, write "undefined."

7) P(12, 16); find csc θ. 　　7) D

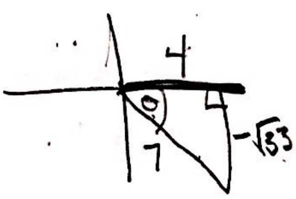
A) $\frac{3}{4}$　$\sin\theta = \frac{16}{20}$　　B) $\frac{4}{3}$　　C) $\frac{5}{3}$　　D) $\frac{5}{4}$

$\to \frac{20}{16} = \frac{5}{4}$

Evaluate without using a calculator.

8) sin θ, if $\cos\theta = \frac{4}{7}$ and $\tan\theta < 0$ 　　8) B

A) $-\frac{7}{4}$　　B) $-\frac{\sqrt{33}}{7}$　　C) $-\sqrt{33}$　　D) $-\frac{\sqrt{33}}{4}$



$4^2 + x^2 = 7^2$

$16 + x^2 = 49$

$\sqrt{x^2} = \sqrt{33}$

$(-1,0)$

Evaluate the trigonometric function of the given quadrantal angle. If the value is undefined, write "undefined."

9) $\cos 3\pi$ "x"

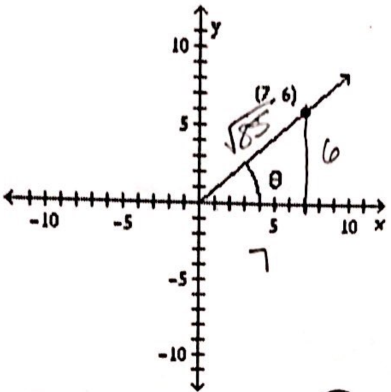
A) -1 B) 1 C) undefined D) 0

9) A

Find the trigonometric function value for the angle shown.

10) $\sec \theta$

10) A



$7^2+6^2=c^2$

$49+36=c^2$

$\sqrt{85}=\sqrt{c^2}$

$\cos\theta = \frac{7}{\sqrt{85}}$

$\rightarrow \frac{7}{\sqrt{85}} \cdot \frac{\sqrt{85}}{\sqrt{85}}$

$= \frac{7\sqrt{85}}{85}$

$\sec\theta = \frac{\sqrt{85}}{7}$

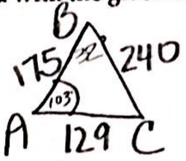
A) $\sec \theta = \frac{7\sqrt{85}}{85}$ B) $\sec \theta = \frac{\sqrt{85}}{7}$ C) $\sec \theta = \frac{7}{6}$ D) $\sec \theta = \frac{6}{7}$

Decide whether a triangle can be formed with the given side lengths. If so, use Heron's formula to find the area of the triangle.

11) a = 240, b = 129, c = 175

A) 6326.96 B) 10,987.87
C) No triangle is formed. D) 6312.86

11) B



$240^2=129^2+175^2-2(129)(175)\cos A$

$57600=47266-45150\cos A$

$10334=-45150\cos A$

$-.22888=\cos A$

$A=103$

$\frac{240}{\sin 103} = \frac{129}{\sin B}$

$\frac{240\sin B = 129\sin 103}{240}$

$\sin B = .5237$

$\angle B: 32°$

or ~~148°~~

$\boxed{1\Delta}$

$A=\frac{1}{2}(175)(129)\sin 103 = 10998$

Solve the problem.

$\frac{28}{60}=\frac{7}{15}$ of $2\pi = \frac{14\pi}{15}$

12) The minute hand of a clock is 13 inches long. What distance does its tip move in 28 minutes?

A) $\frac{91}{15}\pi$ in. B) $\frac{14}{195}\pi$ in. C) $\frac{182}{15}\pi$ in. D) $\frac{7}{195}\pi$ in.

12) C

28 min
13

$S=\theta r$ $S=\frac{14\pi}{15}\cdot 13 = \frac{182\pi}{15}$

State whether the given measurements determine zero, one, or two triangles.

13) A = 53°, a = 23, b = 28

A) One B) Zero C) Two

13) C

B
23
53
A 28 C

$\frac{28}{\sin B} = \frac{23}{\sin 53}$

$\frac{28\sin 53 = 23\sin B}{23}$

$B = 76°$ or $104°$

$53°$ $53°$

$51°$ $23°$

14) C = 30°, a = 34, c = 17

A) Two B) One C) Zero

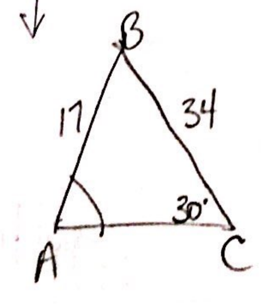
14) B

Solve the triangle.

15) A = 42°, B = 30°, b = 8

A) C = 108°, a ≈ 6, c ≈ 15.3 B) C = 108°, a ≈ 6, c ≈ 11.4
C) C = 108°, a ≈ 10.7, c ≈ 15.3 D) C = 18°, a ≈ 6, c ≈ 11.4

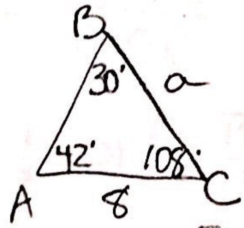
15) C



$\frac{34}{\sin A} = \frac{17}{\sin 30°}$

$\frac{34\sin 30° = 17\sin A}{17}$

$\sin A = 1$ $\angle A = 90°$



$\frac{8}{\sin 30} = \frac{a}{\sin 42}$

$\frac{8\sin 42 = a\sin 30}{\sin 30}$

$10.7 = a$

$\frac{c}{\sin 108} = \frac{8}{\sin 30} \approx 15$

*16. Find the period of $y = -4\cot(2x)$ $\cancel{\frac{2\pi}{B}}$

(A) π (B) $\frac{\pi}{2}$ (C) 2π (D) $\frac{\pi}{4}$ (E) None of these

17. Simplify the expression $\frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{1 \cdot \cos^2\theta}$ completely. The result is

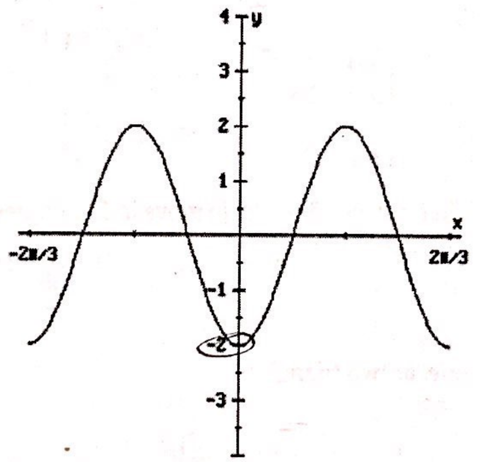
$$\frac{\sin^2\theta \;\; \cancel{1-\cos^2\theta}}{\cos^2\theta}$$

(A) $\cot^2\theta$ (B) $\sec^2\theta$ (C) 0 (D) $\tan^2\theta$ (E) None of these

18. Simplify $\frac{\sec x}{\csc x}$ completely. $\frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} = \frac{1}{\cos x} \cdot \frac{\sin x}{1} = \frac{\sin x}{\cos x}$

(A) 1 (B) $\tan x$ (C) $\tan^3 x$ (D) $\cot x$ (E) $\cot^2 x$

19. Which of the following could be the equation of the function $g(x)$ graphed below?



$P = \frac{2\pi}{3}$

$B = \frac{2\pi}{\frac{2\pi}{3}} = \frac{2\pi \cdot 3}{2\pi} = 3$

(A) $g(x) = -2\cos(3x)$ (B) $g(x) = -2\sin(3x)$ (C) $g(x) = -2\cos\left(\frac{x}{3}\right)$

(D) $g(x) = -2\sin\left(\frac{x}{3}\right)$ (E) $g(x) = 2\sin(3x)$

20. Let θ be an angle in standard position. The terminal side of θ intersects the unit circle at $\left(-\frac{2}{5}, \frac{\sqrt{21}}{5}\right)$. Find $\cot\theta$. $\frac{\cos\theta}{\sin\theta} = \frac{x}{y}$ $\frac{\frac{-2}{5}}{\frac{\sqrt{21}}{5}} = \frac{-2}{5} \cdot \frac{5}{\sqrt{21}} = \frac{-2}{\sqrt{21}}$

(A) $\sqrt{21}$ (B) $\frac{\sqrt{21}}{5}$ (C) $-\frac{1}{5}$ (D) $-\frac{2}{\sqrt{21}}$ (E) $-\sqrt{21}$